Efficient Panel Method for Vortex Sheet Roll-Up

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In order to improve the accuracy of a potential flow three-dimensional wing panel method, wake deformation must be taken into account. This leads to a better knowledge of the large-scale flow behavior. In this paper, a computational method modeling the vortex sheet behind a wing is presented. The sheet deformation is computed using a two-dimensional unsteady analogy in the crossflow plane. The local velocities are induced by linear vorticity panels with smoothed velocity fields. The sheet is rediscretized at each time step with an adaptive curvature-dependent paneling scheme. The vortex sheet model produces good results for the typical cases of elliptic loading, wing/flap configuration, and ring wing. Coupling of the wake model with a three-dimensional wing panel method gives wake geometries that compare well with those obtained with other codes.

Nomenclature

b = wing span
F = fraction of panel length (displacement factor)
K = ratio of increase or decrease in panel length during rediscretization
L = panel length
$\boldsymbol{r}$ = position vector of a panel differential element relative to a point in the flow field
s = length of the sheet from the plane of symmetry
t = time-like coordinate (related to the distance from the trailing edge)
U = velocity vector
Uy, Uz = lateral and vertical velocity components
x = position along the longitudinal axis (zero at the trailing edge)
y = position along the lateral (spanwise) axis (zero in the plane of symmetry)
z = position along the vertical axis (zero in the plane of the wing)
Γ = bound circulation of the wing (equivalent to the doublet jump)
Δt = time step
Δy, Δz = lateral and vertical displacements
δ = smoothing factor
Θ = angle between a panel and the positive spanwise y-axis
μ = angle change between two successive panels
Ω = vorticity vector

Subscripts
i, n = panel number
max = maximum allowed
∞ = in the free-stream
0 = reference value (parameter)

Introduction

PANEL methods routinely evaluate the incompressible flowfield around complex 3D configurations. Extensively used in the design of new aircraft, they are relatively easy to implement because they are based on linear equations. In such methods, the shape of the wake must be known a priori, and simplifying assumptions generally lead to the use of a flat vortex sheet. However, a better estimation of the sheet configuration can lead to an improved solution, with a better prediction of downwash and sidewash. Computing the correct geometry of the wake behind lifting surfaces involves nonlinear effects that can only be included by using a relaxation scheme in which the vortex sheet shape is iteratively modified to become a stream surface. Most full-scale panel methods use many grid points on the surface of the wake and displace them along the local velocity vector to get an improved estimation of the geometry.[1,2] These methods require large computational resources since a correct three-dimensional representation of the wake roll-up demands many iterations and grid points. Simpler two-dimensional methods have been used for years to approximate the inviscid wake behind a lifting surface. They model the vortex sheet as an evolving two-dimensional curve in the crossflow plane.

Many researchers have worked on two-dimensional vortex sheet methods. Pullin[3] proposed a similarity solution to describe the initial roll-up stages of a semi-infinite vortex sheet. Westwater[4] used a small number of discrete vortices to model the wake and compute the local velocities; however, his method presents numerical instabilities when the number of vortices is increased to improve precision. Fink and Soh[5] found that this numerical instability was a result of accumulated logarithmic errors that could be controlled by rediscretizing the vortex sheet into equidistant discrete vortices at each time step. Their method works for a limited distance behind the wing before producing unphysical results, i.e., the vortex sheet crosses itself. Maskew[6] obtained similar results with his subvortex technique. In this technique, when the velocity must be computed in the neighborhood of a discrete vortex, that vortex is subdivided into smaller vortices along the curve of the sheet. A different method called cloud in cell has been used by Baker[7] to describe vortex sheet roll-up; this method uses a rectangular grid and produces grid-dependent small scale substructures that are not always observed experimentally.

Hoeijmakers and Vaatstra[8] presented a complex second-order panel method capable of describing complicated vortex-sheet motion with curved panels. His method gives excellent results but requires almost as much computational resources as a fully three-dimensional method. It is argued here that a simpler first-order panel method is better suited for the pre-

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diction of the wake shape behind a wing analyzed with a simple three-dimensional wing panel method. The adaptive curvature-dependent paneling scheme put forward by Hoeijmakers and Vaatstra[8] provides good definition with fewer nodes. Krasny[9] described the wake roll-up through the use of discrete vortex blobs, which are, in fact, ordinary vortices with a modified velocity field to prevent the apparition of infinite velocities at the singular points of the flowfield. His method can track the wake shape for long periods of time without any numerical instability, but requires a large computer since a lot of nodal points are used. Nagati, et al.[10] developed a complex three-dimensional wake method using parametric bicubic patches to describe the geometry. The results are good only for a short distance behind the wing, and the computational requirements are quite extensive. Within these limits, their method is sufficiently powerful to make iterations unnecessary in most cases.

In this paper, a simple first-order panel method is used to compute the local velocities responsible for the sheet deformation in the crossflow plane. An adaptive paneling scheme is used to rediscretize the sheet at each time step, and discrete vortices model the rolled-up portions of the sheet. Predicted wake shapes for typical wing loadings are quite good. The method was developed to interface with a three-dimensional wing panel method.

Formulation

Wake roll-up behind a finite span wing is fundamentally a three-dimensional phenomenon. In order to simplify the problem to the study of an unsteady two-dimensional curve in the crossflow plane, the rate of change of vorticity and geometry along the longitudinal (freestream) direction must be small compared to the lateral (spanwise) direction. This is generally true for wings with an aspect ratio >3 placed at sufficiently low incidence to prevent leading-edge separation. The evolution of the wake with growing distance from the trailing edge is then analogous to the self-induced deformation of an infinitely long vortex sheet with time, the initial shape of the sheet being that of the trailing edge. This unsteady analogy uses a time-like coordinate $t = x/U_\infty$ that locates the crossflow plane in which the wake is being modeled (Fig. 1).

The vortex-sheet approach used here is justifiable at very large Reynolds number, where the real wake is very thin and viscous forces are negligible. The method was developed for incompressible flow. It assumes a symmetrical geometry, therefore permitting the use of a half-span model. The input for this method has two components: the shape of the trailing edge, which gives the initial shape of the two-dimensional unsteady vortex sheet, and the spanwise wing loading, given as the jump in doublet strength at the trailing edge. This doublet distribution Γ is simply convected away by the stretching vortex sheet because the vorticity cannot change away from the body in potential flow. The problem then reduces to solving an initial-value problem (Fig. 2).

From Prandtl lifting-line theory, the negative of the vorticity shed into the wake is given by the derivative of the doublet distribution:

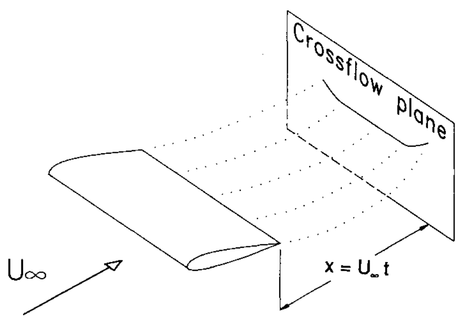


Fig. 1 Definition of the time-like coordinate locating a given crossflow plane in the two-dimensional unsteady analogy.

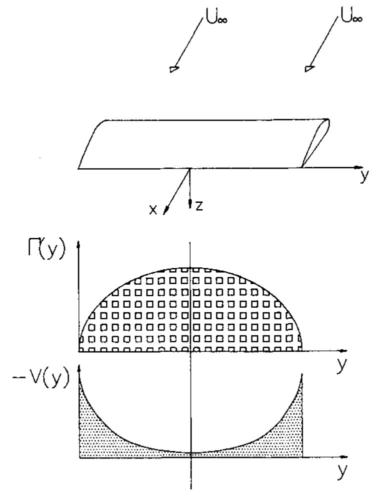


Fig. 2 Initial conditions on the wake shed at the trailing edge.

$$\Omega(s) = -\frac{d\Gamma}{ds} \tag{1}$$

A typical vortex-sheet panel is described by the coordinates of its endpoints and the vorticity associated with each of its ends. In order to simplify the rediscretization process and the parametric spline representation, the doublet values and coordinates of the endpoints are stored as functions of the sheet length s. The velocity field induced by such a panel with linear vorticity is found in closed form by integration of the Biot-Savart law[11]:

$$\boldsymbol{U} = \int_{s_i}^{s_{i+1}} \frac{\boldsymbol{r} \times \Omega(s)}{2\pi r^2} \, ds \tag{2}$$

To make the velocity field more regular and to control the numerical Kelvin-Helmoltz instability that is so common in vortex methods, a nondimensional smoothing factor δ is introduced, as proposed by Krasny.[9] The effect of this smoothing is to modify slightly the Biot-Savart law to remove the possibility of infinite velocity:

$$\boldsymbol{U} = \int_{s_i}^{s_{i+1}} \frac{\boldsymbol{r} \times \Omega(s)}{2\pi(r^2 + b^2\delta^2)} \, ds \tag{3}$$

The introduction of smoothing in the velocity field is similar to building the vortex panel from distributed vortex blobs (see Ref. 12 for a review of computational methods with vortices). The normal velocity induced by a panel on itself is continuous across the panel, but the tangential velocity is discontinuous when the smoothing factor tends toward zero. The smoothing factor introduces a gradual change in tangential velocity. This provides a better approximation to the real, viscous flow and stabilizes the numerical scheme.

Highly Rolled-Up Vortex Sheet Modeling

One of the problems encountered by wake relaxation programs is that the number of required nodes keeps growing as the sheet stretches while rolling up. This is particularly true when a curvature-dependent paneling scheme is used. Therefore, a simpler way of modeling the highly rolled-up portion is needed. In the method presented here, a maximum Θ_{max} is set for the angle of roll-up (Fig. 3), and panels going beyond that maximum are cut to create a concentrated vortex, either the so-called tip vortex at the end of the sheet or a double-branched vortex in the middle of it. (In a vortex sheet, a double-branched vortex is a spiral structure linked to the re-

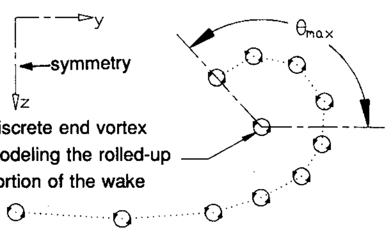


Fig. 3 Modelization of the highly rolled-up portion of the vortex sheet.

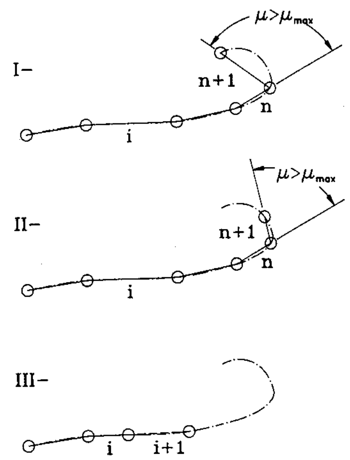


Fig. 4 Adaptive discretization of a curved sheet.

mainder of the sheet by two arms.) Such a discrete vortex generates a velocity field that externally resembles that of the highly rolled-up region it replaces, as discussed by Smith.[13] Closer to the discrete vortex, the velocity field is kept finite by using the same smoothing factor used for the vortex panels, in essence creating a vortex blob that corresponds to a small volume of rotational flow.

Adaptive Paneling Scheme

The curvature of a deforming vortex sheet varies widely, and it is one of the goals of the computation to obtain some details about the flow in the highly curved regions. In those regions, discretizing the sheet with straight panels will result in considerable errors if the panel length is not small enough. However, using such small panels in relatively flat regions unnecessarily increases the number of points required and, thus, the computer costs. Therefore, the panel length must vary. The constraints put on the discretization procedure are that the angle change μ should be smaller than a specified maximum μ_{max} and that the panels should be as large as possible without exceeding a given fraction of the total sheet length.

The procedure used starts with the maximum length for the panel at the plane of symmetry and then proceeds toward the tip. This procedure tries to make the length L_{n+1} of a new panel larger than its predecessor by a fixed ratio K, $L_{n+1} = KL_n$, without exceeding the maximum. If the angle change μ is then seen to exceed μ_{max}, the panel is instead made smaller than its predecessor, $L_{n+1} = L_n/K$. If this still yields $\mu > \mu_{max}$, the procedure goes back to previous panels until it finds a panel, labeled i, that is not smaller than the preceding one, i.e., $L_i > L_{i-1}/K$. This panel is then shortened to $L_i = L_{n-1}/K$ and the procedure starts again with $n = i + 1$ (Fig. 4). The process is recursive, but has been found to be quite fast, even for convoluted sheets. Typically, the ratio K is around 1.4.

Description of the Method

As stated earlier, the doublet value at each node is conserved during the evolution of the wake; it can be shown that this corresponds to Helmholtz's theorem of vortex continuity. Once the initial doublet distribution is known, the method itself may proceed with the following steps:

1) The local velocities at each panel midpoint are computed by summing the contributions of each wake panel, of the concentrated double-branched vortices (if any), and of the concentrated end vortex.

2) The panel midpoints are displaced using a simple Euler scheme:

$$\Delta y_i = \Delta t \cdot Uy_i \tag{4a}$$

$$\Delta z_i = \Delta t \cdot Uz_i \tag{4b}$$

The displacements of both sheet extremities are then extrapolated from those of the nearest midpoints. The time step Δt is computed at each step. It is chosen small enough to ensure that no panel midpoint moves by more than a specified fraction F of the length of the panel. The fraction F is a parameter of relatively small value; it is typically around 0.125. It controls the precision and the computing time required, a better precision calling for more computational steps.

3) The displacement of the nodes (panel endpoints) are then interpolated from the midpoints displacements using

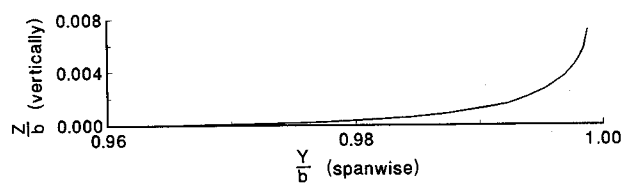


Fig. 5 Pullin's self-similar solution for the outboard 5% of the wake behind an elliptically loaded wing.

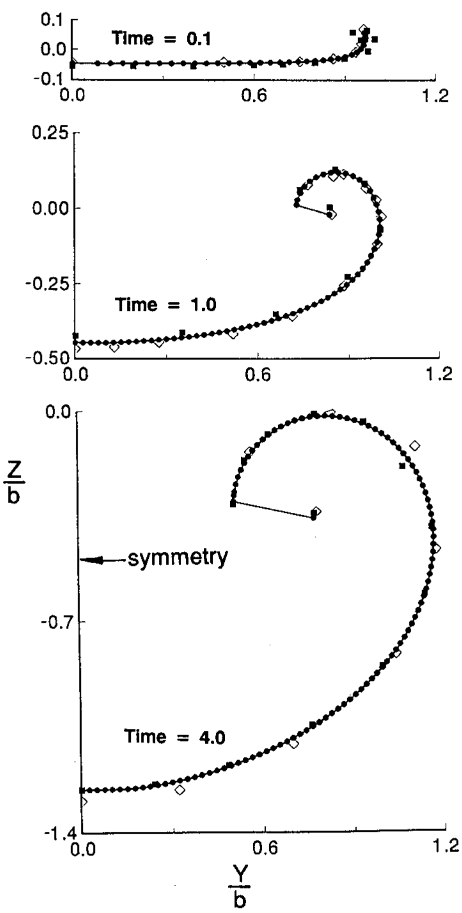


Fig. 6 Wake geometry behind an elliptically loaded wing for timelike coordinates of t = 0.1, 1.0, and 4.0: ■ from Hoeijmakers and Vaatstra[8]; ◇ from Krasny.[9]

splines. The discrete end vortex is also displaced by a distance corresponding to the time step times the local velocity.

4) The angle of rotation Θ of each panel is computed, starting with the panel at the plane of symmetry and proceeding toward the end in such a way as to yield angles of more than 2π when more than one turn of roll-up has occurred. If a panel has a rotation angle exceeding Θ_{max}, the maximum allowed, the panel is cut and its vorticity is added to that of a concentrated vortex. If the panel angle is only a local maximum, a double-branched vortex is created; otherwise, the panel is compound with the end vortex at the center of vorticity of the old end vortex and the newly cut panel.

5) Cubic spline representations of the coordinates (y_i, z_i) are computed using the sheet length s as a parameter. The doublet values associated with the nodes do not change since the doublet is simply convected, but the stretching of the wake makes it necessary to update the parametric spline representation of the doublet distribution.

6) The vortex sheet is rediscretized using the cubic spline representations and the adaptive paneling procedure. Typically, μ_{max} is chosen so that 10–20 panels are required to model a complete turn.

7) The doublet value associated with each new panel endpoint is computed from the spline representation.

Steps 1–7 are repeated until the computation has covered a specified distance behind the trailing edge. When the method is coupled to a three-dimensional panel method, the results must be interpolated to give the positions of the continuous vortex filaments originating from the wing; the output in this case consists of the changing positions, in the crossflow plane, of the points having the same doublet values as those of the vortices initially shed at the trailing edge. In the computer code used, no far-field vortex agglomeration was performed. It is believed that this technique would lower the required computer time, especially in the case that the number of nodal points becomes large.

Elliptically Loaded Wing

The typical case of an elliptically loaded wing has been used extensively to compare wake relaxation programs. The spanwise doublet distribution is then

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{y}{b}\right)^2} \tag{5}$$

In this case, however, the infinite vorticity at the wingtip causes numerical problems and is eliminated by starting the computation with a part of the self-similar roll-up solution of Pullin.[3] However, this solution holds only for a semi-infinite vortex sheet. It is, therefore, applied only to the outermost 5% of the emitted wake, the remaining part of the sheet being straight. From the nondimensional parameters of Pullin's analysis, it can be shown that the self-similar solution applies to this small portion of the wake when the time-like coordinate t = 0.00271. From this initial condition (Fig. 5), the wake evolution is computed using the steps outlined before.

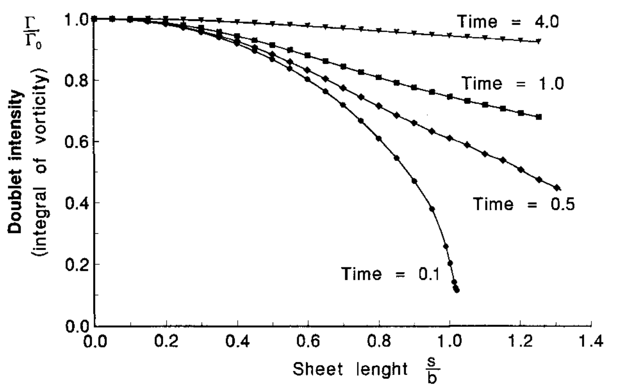


Fig. 7 Doublet distribution on the vortex sheet behind an elliptically loaded wing.

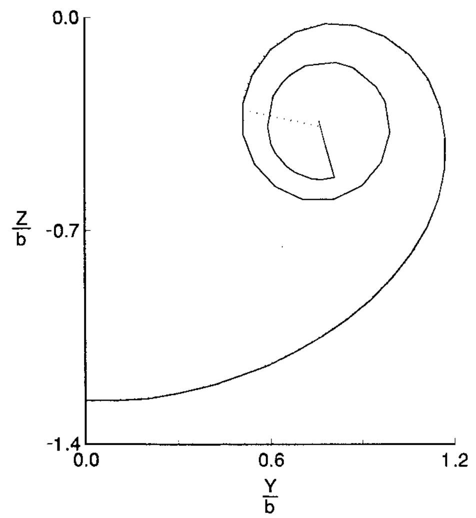
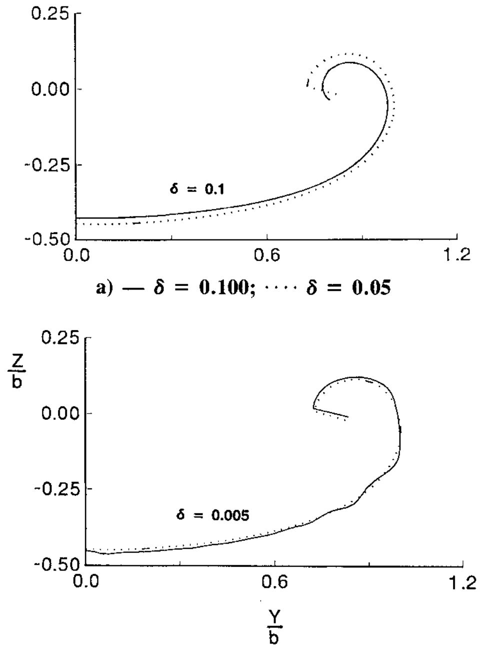


Fig. 8 Wake geometry at t = 4.0: — 720 deg of roll-up; ···· 270 deg of roll-up.



a) — δ = 0.100; ···· δ = 0.05

b) — δ = 0.005; ···· δ = 0.05

Fig. 9 Wake geometry at t = 1.0.

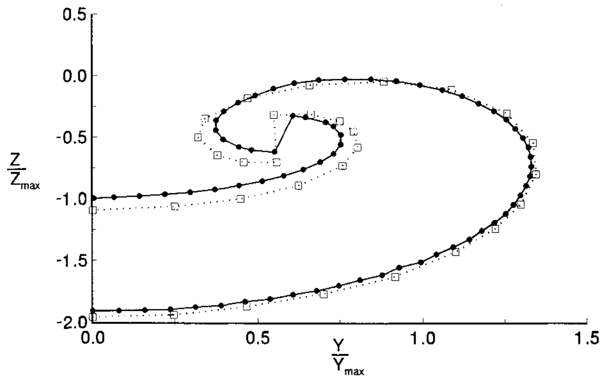


Fig. 10 Wake behind a ring wing at t = 3.0: ···· from Hoeijmakers and Vaatstra.[8]

Results were obtained with $\delta = 0.05$ for time-like coordinates of $t = 0.1$, 1.0, and 4.0 (Fig. 6). The computation took less than 20 min on the IBM 3090 and no more than 65 panels were required at any given time. The maximum panel length was 2.5% of the total sheet length and the roll-up was allowed to be as large as 270 deg. As can be seen in the figure, the results compare very well to those obtained by Hoeijmakers and Vaatstra[8] and Krasny.[9] For time-like coordinates of 1.0 and 4.0, the three methods predict the same position for the tip vortex. It must be noted that, when the time-like coordinate reaches $t = 4.0$, the tip vortex contains 92% of the total vorticity (Fig. 7); the remaining stages of evolution are correctly predicted by taking into account only the effects of the tip vortex and its mirror image. The validity of cutting the rolled-up portion of the sheet to keep less than a turn of roll-up is demonstrated in Fig. 8, which shows that the results are exactly the same for a cut made after two complete turns of roll-up. Similarly, halving the number of nodes (maximum panel length of 5.5% of the total sheet length) gave similar results in one-half the computer time.

The effect of the smoothing factor on the solution was also investigated. As anticipated, a larger smoothing factor slightly slowed the roll-up process, whereas a very small amount of smoothing gave rise to Kelvin-Helmoltz instability (Fig. 9). With no smoothing at all, the solution is very unstable[11] and small double-branched vortices rapidly appear all along the vortex sheet. This behavior practically halts the progress of the computation behind the trailing edge since the time step is proportional to the length of the panels. This length becomes very small when the adaptive paneling scheme attempts to obtain a good definition of the resulting short waves. As outlined by Krasny, round-off errors play an important role in this instability. Smoothing the velocity field therefore ensures results that are more repeatable.

Ring Wing

In order to test the wake model's ability to capture and track double-branched vortices, the ring wing case was studied. The shed vorticity is regular in this case, and no starting solution is required. At the trailing edge, the singularity distribution and the geometry are described by

$$\Gamma(s) = \Gamma_0 \cos\left(\frac{\pi s}{s_{\max}}\right) \tag{6a}$$

$$Y(s) = Y_{\max} \sin\left(\frac{\pi s}{s_{\max}}\right) \tag{6b}$$

$$Z(s) = -Z_{\max} \cos\left(\frac{\pi s}{s_{\max}}\right) \tag{6c}$$

with $s = 0$ at the bottom and $s = s_{\max}$ at the top.

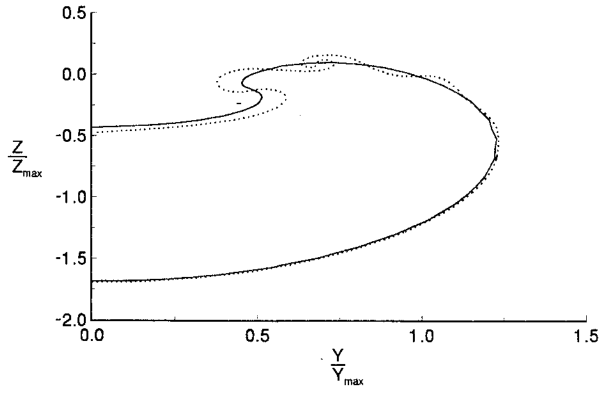


Fig. 11 Wake behind a ring wing at $t = 2.0$: — $\delta = 0.05$; ···· $\delta = 0.025$.

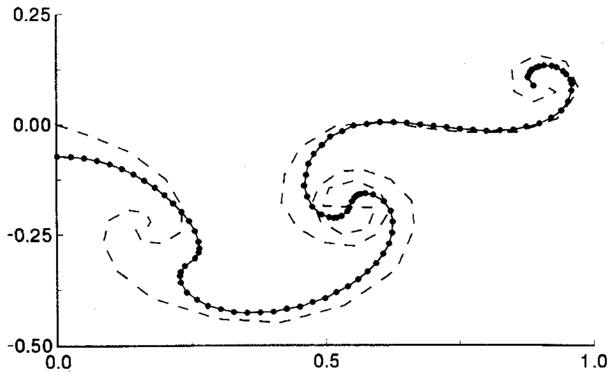


Fig. 12a Wake geometry behind the wing/flap configuration at $t = 0.3$ and $\delta = 0.025$: ---- from Hoeijmakers and Vaatstra.[8]

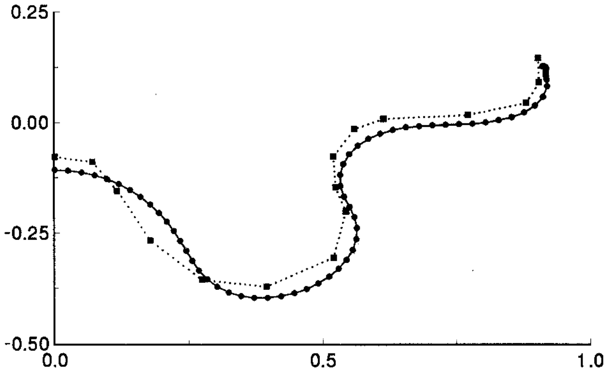


Fig. 12b Wake geometry behind the wing/flap configuration at $t = 0.3$ and $\delta = 0.1$: ···· from Krasny.[9]

The computations were carried out with the same parameters used in the basic elliptic case, except that the maximum angle of roll-up was 360 deg. The parameter $\mu_{\max}$ was chosen so that 16 panels would be necessary to describe a tightly wound circle. Between 40 and 76 panels were used to describe the sheet during the computations. The results are presented in Fig. 10 and compared with those obtained by Hoeijmakers and Vaatstra.[8] Figure 11 shows that halving the smoothing parameter to $\delta = 0.025$ increases the propensity of the wake to develop double-branched vortices. This suggests that the amount of smoothing should be related to the Reynolds number since the smoothing parameter seems to play the damping role of viscous forces.

Wing with Part-Span Flap and Fuselage

Modeling the wake behind a complex configuration with flaps is a matter of interest because this is the configuration in which the possible ill effects of the wake on the control surfaces are more likely to occur. For an idealized configuration, the singularity distribution on the straight wing is approximated by continuous cubics satisfying[8–10]

$$\Gamma(0) = 1.4 \qquad \Gamma'(0) = 0$$

$$\Gamma(0.3) = 2 \qquad \Gamma'(0.3) = 0$$

$$\Gamma(y > 0.7) \text{ is elliptic} \tag{7}$$

The results presented in Fig. 12a were computed with a smoothing factor $\delta = 0.025$, a displacement factor $F = 0.125$, and a maximum angle of roll-up of 270 deg. Between 48 and 100 panels were necessary during the 65 min it ran on the IBM 3090. The results show the same features as those obtained by Hoeijmakers and Vaatstra,[8] even though the angle of roll-up is different. With a smoothing factor $\delta = 0.1$, the results of Fig. 12b compare very well to those of Krasny.[9]

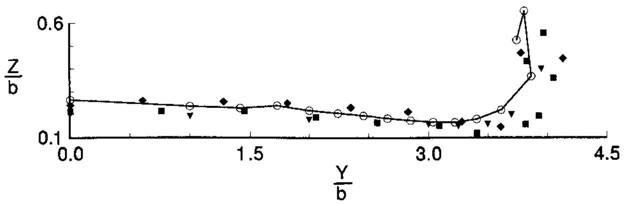


Fig. 13 Comparison of the computed wake geometry behind a rectangular wing of $AR = 8$ at 5 deg of incidence: ■ from Suciu and Morino[15]; ♦ from Yeh and Plotkin[16]; ▼ from VSAERO.

Once again, the smoothing parameter seems to be controlling the complexity and the definition of the computation.

Coupling with a Three-Dimensional Wing Panel Method

The method presented in this paper is to be used in conjunction with a three-dimensional wing panel method to predict the potential flowfield around a three-dimensional configuration. The wake model was therefore tested with the discretized output of such a three-dimensional method[14] for a rectangular wing of aspect ratio 8 at 5 deg of incidence. Only 70 s of computer time were required. The results obtained are shown in Fig. 13, along with those of three other codes: a commercial three-dimensional panel method (VSAERO), a three-dimensional finite element code,[15] and a first-order three-dimensional panel method of wake relaxation.[16] Figure 13 shows that the shape is correctly predicted but that the rolled-up end portion of the curve is located vertically higher. A small bump is also visible in the middle of the sheet. It is probably a result of the Kelvin-Helmoltz instability. The shape illustrated in Fig. 13, which is to be used as new input in the three-dimensional wing method, is made up of only 16 points. These points have the same doublet values as the 16 nodes of the initial output from the three-dimensional wing method. However, the vortex sheet roll-up is tracked with up to 54 points in the wake procedure for improved accuracy.

Conclusions

A first-order panel method has been developed to accurately compute the self-induced deformation in the crossflow plane of the vortex sheet created by a lifting wing. The tendency of the inviscid fluid to develop Kelvin-Helmoltz instability waves is controlled through the use of a smoothing factor. Comparison with experiments of various scales are necessary to establish a correlation between the smoothing factor and the Reynolds number.

The panel method has demonstrated its ability to describe the vortex sheet evolution with fewer nodal points and, thus, less computational resources than other methods. The good stability characteristics obtained with adequate smoothing permit relatively large time steps without compromising accuracy. The method keeps track of all the relevant sheet structures with an adaptive paneling procedure and it models highly rolled-up portions of the sheet with discrete vortices to save computer time and memory. Computational expense is the only limitation on the complexity of the problems that can be treated. By coupling it with a three-dimensional potential flow wing panel method, the vortex sheet roll-up method has proven its utility in predicting the correct wake shape behind a complete three-dimensional wing.

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